Journal of Engineering Physics and Thermophysics, Vol. 77, No. 6, 2004

HYDRODYNAMICS AND HEAT EXCHANGE IN DISPERSED FLOWS

MIXING OF PARTICLES IN APPARATUSES WITH A CIRCULATING FLUIDIZED BED

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UDC 533.6

A phenomenological model of mixing of particles in a circulating fluidized bed has been formulated; a distinctive feature of the model is allowance for convective particle fluxes in the radial direction that ensure a substantial decrease observed in practice in the concentration of the particles over the riser's height. As a result of a comparison of experimental and calculated mixing curves it has been established that the value of the coefficient of radial dispersion of the particles lies in the interval 0.0006–0.006 m^2/sec.

It is common knowledge that the mixing of particles in a circulating fluidized bed (CFB) is a multifactor process occurring against a background of a complex internal hydrodynamics [1]. To model the phenomenon one has used different schemes, which reflect, to an extent, the regularities of the actual process of mixing of particles in CFB apparatuses. The models proposed can arbitrarily be subdivided into two large groups. The first group includes the models of axial (longitudinal) mixing, in which the determining quantities and concentrations are considered as being dependent only on the longitudinal component. In [2], a critical analysis of such models has been made and a fairly simple two-zone model of longitudinal mixing has been proposed on the basis of such analysis; the model includes the following equations:

for the bed's core

$$\frac{\partial A\rho_1\bar{c}_1}{\partial t} + u_1\frac{\partial A\rho_1\bar{c}_1}{\partial x} = \beta_*\rho\,(\bar{c}_2 - \bar{c}_1) - A\rho_1\beta_1\bar{c}_1\,, \qquad (1)$$

for the annular zone

$$\frac{\partial B\rho_2\bar{c}_2}{\partial t} - u_2\frac{\partial B\rho_2\bar{c}_2}{\partial x} = \beta_*\rho\,(\bar{c}_1 - \bar{c}_2) + A\rho_1\beta_1\bar{c}_1\,. \qquad (2)$$

As has been shown in [2], this model is capable of satisfactorily describing experimental mixing curves obtained for both the core of the bed and its annular zone.

Among the second group are the models in which consideration is given to the transfer of tagged particles in both the axial and radial directions. In [3], the experimental data have been interpreted on the basis of the two-dimensional nonstationary model

$$\frac{\partial c^*}{\partial t} = D_{\mathrm{a}}\frac{\partial^2 c^*}{\partial x^2} + \frac{D_{\mathrm{r}}}{r}\frac{\partial}{\partial r}\left(r\frac{\partial c^*}{\partial r}\right) - \frac{\partial\,(v_s c^*)}{\partial x}\,, \qquad (3)$$

which disregards the convective motion of particles from the core of the bed to its annular zone. Therefore, it seems to be applicable only in the upper part of a CFB, where the mechanism mentioned is attenuated and the concentrations of particles in the two zones are independent of height, in practice.

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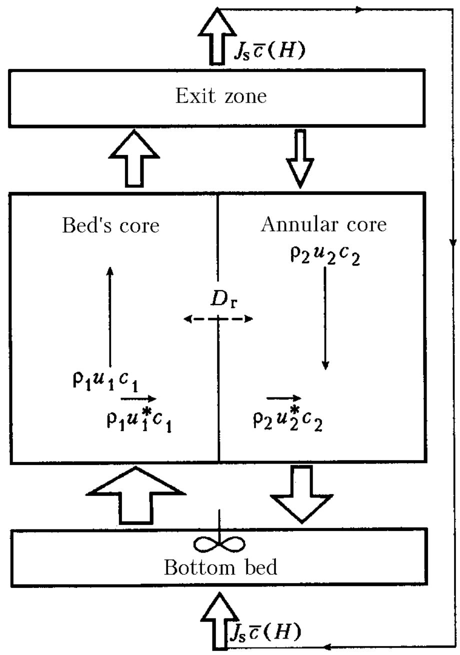

Fig. 1. Model of mixing of particles in a CFB.

Variation in the density over the CFB height is also disregarded in the model of [4]:

$$\frac{\partial(\rho(r)c)}{\partial t}+\frac{\partial(J_s(r)c)}{\partial x}=\frac{D_r}{r}\frac{\partial}{\partial r}\left(r\frac{\partial(\rho(r)c)}{\partial r}\right), \tag{4}$$

where $J_s(r)$ is the local mass particle flux (positive in the bed's core and negative in the annular zone). Model (4) reflects the two-zone CFB structure and, just as (3), can justifiably be used only in the upper part of the bed. Furthermore, because of the incorrect representation of the dispersion term for $t \to \infty$, when $c \to c_\infty$ = const, it yields a physically absurd expression: $\frac{c_\infty D_r}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\rho(r)}{\partial r}\right) \neq 0$. Nonetheless, as Patience and Chaouki [4] report, this model satisfactory described experimental data in the upper part (x = 4 m) of a five-meter riser.

The range of application of models (3) and (4) is seen to be limited. They are applicable only in the upper zone of a CFB, where ρ = const. Furthermore, it is significant that allowance for the most important mechanism of mixing — radial convection of particles (Fig. 1) — is absent in explicit form from the models mentioned; the radial convection of particles ensures a substantial (observed in practice) decrease in ρ_1 and ρ_2 with height for virtually constant longitudinal velocities u_1 and u_2.

The present work seeks to allow for the above mechanism of mixing, thus increasing the level of reality of a theoretical analysis, and to develop on this basis a generalized model of the process, describing the mixing of particles throughout the volume of the CFB riser.

The main assumptions used as the basis for the physical model are as follows:

1. In the CFB core, we have the upward motion of particles with a velocity u_1 and the descending motion near the riser's walls with a velocity u_2 (Fig. 1); these velocities are calculated from the formulas

$$u_1=\frac{u}{A}-u_t, \tag{5}$$

$$u_2=0.1\,(u-u_t)\,\mathrm{Fr}_t^{-0.7}. \tag{6}$$

It is noteworthy that (5) is written under the assumption that filtration of the gas is absent from the annular zone, whereas (6) is written from the data of [5]. The velocity of the gas in the bed's core is equal to *u/A* in this case.

2. The local densities of the bed's core ρ_1 and the annular zone ρ_2 depend only on the longitudinal coordinate *x*. Between them we assume the linear relationship

$$\rho_2 = n\rho_1 , \tag{7}$$

where *n* is a coefficient dependent on *x* (see below).

3. In any horizontal cross section, the equality

$$J_s = Au_1\rho_1 - Bu_2\rho_2 , \tag{8}$$

expressing the existence of the external circulation of particles, holds.

4. The average (over the horizontal section) density of the bed is described by the empirical formula [6]

$$\frac{\rho}{\rho_s} = \bar{J}_s (x')^{-0.82} , \quad H_0' < x' \le 1 . \tag{9}$$

5. In the lower part of the layer, we have the zone with a constant density and virtually ideal mixing of particles — the bottom fluidized bed. Particles from the descending contour and the annular zone are accelerated in it. This region of a CFB acts as an unusual kind of dynamic gas distributor whose height depends on the CFB's operating parameters and is calculated from the formula [7]

$$H_0' = 1.25\, \mathrm{Fr}_t^{-0.8}\, \bar{J}_s^{-1.1} . \tag{10}$$

The porosity of the bottom bed weakly depends on the operating conditions of a CFB. In [7], it is proposed that this quantity be calculated from the dependence

$$\varepsilon_{fb} = 1 - 0.33\, \mathrm{Fr}_t^{-0.45} . \tag{11}$$

6. Allowance is made for the radial convection of particles from the bed's core to the annular zone with velocities u_1^* (core) and u_2^* (annular zone) (Fig. 1). It is clear that the values of these velocities are determined by the rate of change of the average density of the CFB over the riser's height (see below).

7. The dispersion transfer of tagged particles in the longitudinal direction is disregarded. The fact that this is possible has been discussed in [2].

8. The coefficient of radial dispersion of particles is the same in the two zones.

As is seen from (7)–(9), to determine the quantities *A*, *B*, and *n* we have only two equations:

$$Au_1' + Bnu_2' = (A + Bn)\, (x')^{0.82} , \tag{12}$$

$$A + B = 1 . \tag{13}$$

It follows that one of these quantities must be determined in advance. In [2], we set *n* = 3 on the basis of experimental data and found the quantities *A* and *B* as functions of *x*. In the present work, for convenience of numerical calculations we have taken them to be constant and equal to *A* = 0.85 and *B* = 0.15 and have determined the quantity *n* as

$$n = \frac{A}{B}\, \frac{u_1' - (x')^{0.82}}{u_2' + (x')^{0.82}} . \tag{14}$$

On the basis of the assumptions made, which are the physical model of the process, we write the system of convection-dispersion equations representing the transfer of tagged particles in the CFB riser:

$$0 \le r < r_0 : \frac{\partial \rho_1 c_1}{\partial t} + u_1 \frac{\partial \rho_1 c_1}{\partial x} + \frac{\rho_1}{r} \frac{\partial}{\partial r} (r u_1^* c_1) = \frac{\rho_1 D_{\mathrm{r}}}{r} \frac{\partial}{\partial r} \left(r \frac{\partial c_1}{\partial r} \right), \tag{15}$$

$$r_0 < r \le R : \frac{\partial \rho_2 c_2}{\partial t} - u_2 \frac{\partial \rho_2 c_2}{\partial x} + \frac{\rho_2}{r} \frac{\partial}{\partial r} (r u_2^* c_2) = \frac{\rho_2 D_{\mathrm{r}}}{r} \frac{\partial}{\partial r} \left(r \frac{\partial c_2}{\partial r} \right). \tag{16}$$

To determine the radial particle velocities u_1^* and u_2^* that ensure a substantial (observed in experiment) decrease in the density of the bed over the riser's height we use the equations of continuity of the particle fluxes in the corresponding zones:

$$0 \le r < r_0 : \frac{\partial \rho_1}{\partial t} + u_1 \frac{\partial \rho_1}{\partial x} + \frac{\rho_1}{r} \frac{\partial}{\partial r} (r u_1^*) = 0 , \tag{17}$$

$$r_0 < r \le R : \frac{\partial \rho_2}{\partial t} - u_2 \frac{\partial \rho_2}{\partial x} + \frac{\rho_2}{r} \frac{\partial}{\partial r} (r u_2^*) = 0 . \tag{18}$$

With account for $\frac{\partial \rho_1}{\partial t} = \frac{\partial \rho_2}{\partial t} = 0$, after the integration of (17) and (18) with respect to r, we obtain the dependences for calculation of the velocities sought:

$$u_1^* = -\frac{r}{2} u_1 \frac{1}{\rho_1} \frac{d\rho_1}{dx} , \tag{19}$$

$$u_2^* = -\frac{AR^2}{2r} \frac{u_1}{\rho_2} \frac{d\rho_1}{dx} + \frac{1}{2} \left(r - \frac{AR^2}{r} \right) \frac{u_2}{\rho_2} \frac{d\rho_2}{dx} . \tag{20}$$

We note that, in derivation of (19) and (20), the equality

$$\rho_1 u_1^* = \rho_2 u_2^* \quad \text{for} \quad r = r_0 , \tag{21}$$

expressing the condition of continuity of the radial particle flux at the boundary of the zones, has been used. As is seen from (19) and (20),

$$u_1^* \big|_{r=0} = 0 , \tag{22}$$

$$u_2^* \big|_{r=R} = -\frac{R}{2\rho_2} \frac{dJ_{\mathrm{s}}}{dx} = 0 . \tag{23}$$

Equalities (22) and (23) and the structure of expressions (19) and (20) demonstrate the physical nature of the dependences obtained for calculation of the velocities u_1^* and u_2^*. The densities of the bed's core and the annular zone involved in (19) and (20) are expressed by the average density of the bed:

$$\rho_1 = \frac{\rho}{A + Bn} , \quad \rho_2 = \frac{n\rho}{A + Bn} . \tag{24}$$

It is easy to show that Eqs. (1) and (2) describing the longitudinal transfer of a tagged impurity in a CFB are a particular case of system (15)–(16). Let us integrate (15) for r going from 0 to r_0 and (16) going from r_0 to R:

$$\frac{\partial A\rho_1\bar{c}_1}{\partial t} + u_1\frac{\partial A\rho_1\bar{c}_1}{\partial x} + \frac{2r_0\rho_1}{R^2}(u_1^* c_1)\big|_{r_0} = \frac{2r_0}{R^2}\rho_1 D_\mathrm{r}\frac{\partial c_1}{\partial r}\bigg|_{r_0}, \tag{25}$$

$$\frac{\partial B\rho_2\bar{c}_2}{\partial t} - u_2\frac{\partial B\rho_2\bar{c}_2}{\partial x} - \frac{2r_0\rho_2}{R^2}(u_2^* c_2)\big|_{r_0} = -\frac{2r_0}{R^2}\rho_2 D_\mathrm{r}\frac{\partial c_2}{\partial r}\bigg|_{r_0}. \tag{26}$$

In writing (25) and (26), we have used equalities (22) and (23) and the conditions of absence of the radial flux of tagged particles on the riser's wall:

$$\frac{\partial c_2}{\partial r}\bigg|_{r=R} = 0\,. \tag{27}$$

In (25) and (26), we formally set

$$\frac{2r_0}{R^2}\rho_1(u_1^* c_1)\big|_{r_0} = A\rho_1\beta_1\bar{c}_1\,, \tag{28}$$

$$\frac{2r_0}{R^2}\rho_1 D_\mathrm{r}\frac{\partial c_1}{\partial r}\bigg|_{r_0} = \beta_*\rho\,(\bar{c}_2 - \bar{c}_1)\,. \tag{29}$$

Relations (28) and (29) express the convection and dispersion fluxes of tagged particles at the boundary of the zones by the average concentrations $\bar{c}_1$ and $\bar{c}_2$ and thereby determine the effective quantities β_1 and β_*, which are the basic parameters of the longitudinal-mixing model (1) and (2).

Equations (1) and (2) follow from (25) and (26) after the substitution of relations (28) and (29) into them with account for the equality

$$\rho_1\left(u_1^* c_1 - D_\mathrm{r}\frac{\partial c_1}{\partial r}\right)\bigg|_{r_0} = \rho_2\left(u_2^* c_2 - D_\mathrm{r}\frac{\partial c_2}{\partial r}\right)\bigg|_{r_0}, \tag{30}$$

which expresses the continuity of the total tagged-impurity flux at the boundary of the zones. We note that this condition can be called a generalized Danckwerts condition.

With the use of the continuity equations (17) and (18), system (15)–(16) is simplified:

$$\frac{\partial c_1}{\partial t} + u_1\frac{\partial c_1}{\partial x} + u_1^*\frac{\partial c_1}{\partial r} = \frac{D_\mathrm{r}}{r}\frac{\partial}{\partial r}\left(r\frac{\partial c_1}{\partial r}\right), \tag{31}$$

$$\frac{\partial c_2}{\partial t} - u_2\frac{\partial c_2}{\partial x} + u_2^*\frac{\partial c_2}{\partial r} = \frac{D_\mathrm{r}}{r}\frac{\partial}{\partial r}\left(r\frac{\partial c_2}{\partial r}\right). \tag{32}$$

The system of equations (31)–(32) in which the values of the axial and radial velocities of particles are prescribed by expressions (5), (6), (19), and (20) reflects the basic aspects of the process of mixing of particles in a CFB and describes the evolution of the corresponding fields.

We consider two limiting cases:

(a) $D_r \to 0$. Equations (31) and (32) will take the form

$$\frac{\partial c_1}{\partial t} + u_1 \frac{\partial c_1}{\partial x} + u_1^* \frac{\partial c_1}{\partial r} = 0 , \tag{33}$$

$$\frac{\partial c_2}{\partial t} - u_2 \frac{\partial c_2}{\partial x} + u_2^* \frac{\partial c_2}{\partial r} = 0 . \tag{34}$$

As is seen, the process of mixing is of a purely convective character in this case.

(b) $D_r \to \infty$. Any difference between the phases is lost. Multiplying (31) by $A\rho_1$ and (32) by $B\rho_2$ and adding together the resulting equations on condition that $c_1 = c_2 = c$, we obtain

$$\rho \frac{\partial c}{\partial t} + J_s \frac{\partial c}{\partial x} = \lim \left(\frac{\rho D_r}{r} \frac{\partial}{\partial r} \left(r \frac{\partial c}{\partial r} \right) \right). \tag{35}$$

Expression $\lim \left(\frac{\rho D_r}{r} \frac{\partial}{\partial r} \left(r \frac{\partial c}{\partial r} \right) \right)$ represents an indeterminacy of the $\infty \cdot 0$ type. The value of this indeterminacy can be found from the condition of physical correspondence of models (1), (2) and (31), (32) for $\beta_* \to \infty$ and $D_r \to \infty$ respectively. It is clear that, under these conditions, the above models must describe the same process. In [2], for (1), (2) when $\beta_* \to \infty$ we obtained the equation

$$\rho \frac{\partial c}{\partial t} + J_s \frac{\partial c}{\partial x} = 0 . \tag{35a}$$

A comparison of (35) and (35a) yields $\lim \left(\frac{\rho D_r}{r} \frac{\partial}{\partial r} \left(r \frac{\partial c}{\partial r} \right) \right) = 0$. Equation (35a) describes the convective transfer of a tagged impurity with a rate J_s/ρ.

The system (31)–(32) obtained was used for mathematical modeling of the mixing of tagged particles introduced into the bottom fluidized bed at the initial instant of time. Let us formulate the corresponding boundary conditions:

the initial conditions

$$c_1 (0, x, r) = c_2 (0, x, r) = 0 , \quad c_1 (0, H_0, r) = c_0 ;$$

the boundary conditions

$$\begin{aligned}
& r = 0 : \quad \frac{\partial c_1}{\partial r} = 0 , \\
& r = R : \quad \frac{\partial c_2}{\partial r} = 0 , \\
& r = r_0 : \quad u_1^* c_1 - D_r \frac{\partial c_1}{\partial r} = n \left(u_2^* c_2 - D_r \frac{\partial c_2}{\partial r} \right), \\
& x = H : \quad \bar{c}_1 = \bar{c}_2 , \\
& \qquad\qquad t \le T : \quad H_0 \rho_{fb} \frac{\partial \bar{c}_1}{\partial t} + A\rho_1 u_1 \bar{c}_1 - B\rho_2 u_2 \bar{c}_2 = 0 , \\
& x = H_0 : \\
& \qquad\qquad t > T : \quad H_0 \rho_{fb} \frac{\partial \bar{c}_1}{\partial t} + A\rho_1 u_1 \bar{c}_1 - B\rho_2 u_2 \bar{c}_2 = J_s \bar{c}_1 (t - \Delta t, H) .
\end{aligned} \tag{36}$$

For more details about the conditions reflecting the influence of the bottom bed and the external circulation of particles when $x = H_0$, see [8]. It is noteworthy that the condition for $x = H$ is a consequence of the relation

$$A\rho_1 u_1 \bar{c}_1 - B\rho_2 u_2 \bar{c}_2 = J_s (A\bar{c}_1 + B\bar{c}_2) , \tag{37}$$

reflecting the presence of the internal circulation of particles and the balance of the tagged-particle fluxes at exit from the riser in the case of a good mixing of the particles in this zone (Fig. 1). It also follows that the quantity c_2 at exit from the riser ($x = H$) will be independent of the radial coordinate: $c_2(t, H, r) = \varphi(t)$. Analogously, the condition of ideal mixing of particles in the bottom bed yields $c_1(t, H_0, r) = \psi(t)$.

Problem (31), (32), and (36) is self-consistent, since the concentration of the tagged particles at entry into the CFB depends on their concentration at exit at the instant of time $t - \Delta t$.

We represent system (31), (32), and (36) in dimensionless form:

$$\frac{\partial c_1}{\partial t'} + u_1' \frac{\partial c_1}{\partial x'} + (u_1^*)' \frac{\partial c_1}{\partial r'} = \frac{1}{\mathrm{Pe}} \frac{1}{r'} \frac{\partial}{\partial r'} \left(r' \frac{\partial c_1}{\partial r'} \right), \tag{38}$$

$$\frac{\partial c_2}{\partial t'} - u_2' \frac{\partial c_2}{\partial x'} + (u_2^*)' \frac{\partial c_2}{\partial r'} = \frac{1}{\mathrm{Pe}} \frac{1}{r'} \frac{\partial}{\partial r'} \left(r' \frac{\partial c_2}{\partial r'} \right). \tag{39}$$

The boundary conditions are

$$c_1 (0, x', r') = c_2 (0, x', r') = 0 , \quad c_1 (0, H_0', r') = c_0 ;$$

$$\begin{aligned}
& r' = 0 : \quad \frac{\partial c_1}{\partial r'} = 0 , \\
& r' = 1 : \quad \frac{\partial c_2}{\partial r'} = 0 , \\
& r' = \sqrt{A} : \quad (u_1^*)' c_1 - \frac{R}{H} \frac{1}{\mathrm{Pe}} \frac{\partial c_1}{\partial r'} = n \left((u_2^*)' c_2 - \frac{R}{H} \frac{1}{\mathrm{Pe}} \frac{\partial c_2}{\partial r'} \right), \\
& x' = 1 : \quad \bar{c}_1 = \bar{c}_2 , \\
& \qquad t' \le T' : \quad H_0' m \frac{\partial \bar{c}_1}{\partial t'} + \frac{u_2' + (H_0')^{0.82}}{u_1' + u_2'} u_1' \bar{c}_1 - \frac{u_1' - (H_0')^{0.82}}{u_1' + u_2'} u_2' \bar{c}_2 = 0 , \\
& x' = H_0' : \\
& \qquad t' > T' : \quad H_0' m \frac{\partial \bar{c}_1}{\partial t'} + \frac{u_2' + (H_0')^{0.82}}{u_1' + u_2'} u_1' \bar{c}_1 - \frac{u_1' - (H_0')^{0.82}}{u_1' + u_2'} u_2' \bar{c}_2 = (H_0')^{0.82} \bar{c}_1 (t' - \Delta t', 1) .
\end{aligned} \tag{40}$$

The dimensionless velocities $(u_1^*)'$ and $(u_2^*)'$ involved in (38)–(40) are calculated from the formulas

$$(u_1^*)' = 0.41 \cdot \frac{r'}{x'} \frac{u_1' u_2'}{u_2' + (x')^{0.82}} , \tag{41}$$

$$(u_2^*)' = 0.41 \cdot \frac{1 - (r')^2}{r' x'} \frac{u_1' u_2'}{u_1' - (x')^{0.82}} , \tag{42}$$

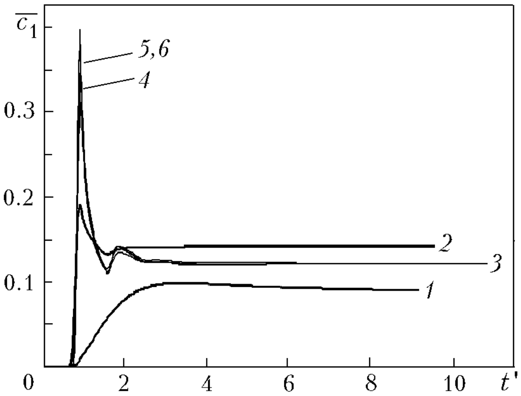

Fig. 2. Average concentration of the impurity at exit from the riser vs. dimensionless time: 1) Pe = 1; 2) 5; 3) 50; 4) 500; 5) 5000; 6) 50,000.

which follow from (19) and (20). The quantity $m = \rho_{fb}/\rho(H_0)$ is determined from the expression $m = 0.4\ \mathrm{Fr}_t^{-0.7}$, which has been obtained from (9)–(11). As is seen, system (38)–(40) contains one unknown parameter — the coefficient of radial dispersion D_r involved in the Pe number.

Let us evaluate the relation of the dispersive and convective radial fluxes of tagged particles α. We can do this using Eq. (38) and formula (41):

$$\alpha \approx \frac{x' \left(u_2' + (x')^{0.82}\right)}{\mathrm{Pe}\, (r')^2\, u_1' u_2'} . \tag{43}$$

With account for D_r = 0.0025 m^2/sec, H = 5 m, R = 0.042 m, u = 8.2 m/sec, and u_t = 2.3 m/sec (experimental conditions in [4]), for $u_1' \approx 1$ and $u_2' \approx 0.2$ we obtain $\alpha \approx \dfrac{6x'(0.2 + (x')^{0.82})}{(r')^2}$. Despite the certain conditionality of the quantity D_r, we can infer that the influence of the radial convective transfer on the process of mixing of particles is substantial (particularly in the lower part of the riser at the boundary of the zones).

The system of unsteady differential equations describing the process of mixing of particles in a circulating fluidized bed was solved numerically by the finite-difference method. We used an implicit finite-difference scheme of first order of accuracy for the spatial step of the computational grid h. The system of difference equations was solved by the marching method.

The size of the spatial step was selected so that the solutions obtained with steps h and $h/2$ differed only in the fourth significant digit. The calculations were stopped upon the establishment of the steady-state regime.

The computational algorithm in passage from one time level to the subsequent level consisted of the following stages:

(1) we calculated the concentration of particles in the bottom bed of the core;

(2) in passage from one spatial level to another with a step h, we computed the concentration distribution of particles in the central part of the bed (core);

(3) we calculated the value of the particle concentration in the upper part of the annular zone;

(4) next, in moving from the top down successively with a step h, we determined the particle concentration in the entire annular zone;

(5) we checked the observance of the condition of the solution reaching the steady-state regime. If it was not observed, the computations were repeated beginning with item 1.

Figure 2 gives the calculation of the average concentrations of tagged particles at exit from the riser for different values of Pe. Here and in what follows the calculations are performed for H = 12 m, u = 6 m/sec, u_t = 0.5 m/sec, ρ_s = 2000 kg/m^3, J_s = 50 kg/(m^2·sec), and Δt = 0 (the tagged material is instantaneously transferred from the point of exit from the riser to the point of reentry for $x = H_0$). The steady-state value of the concentration c_∞ is computed from the formula

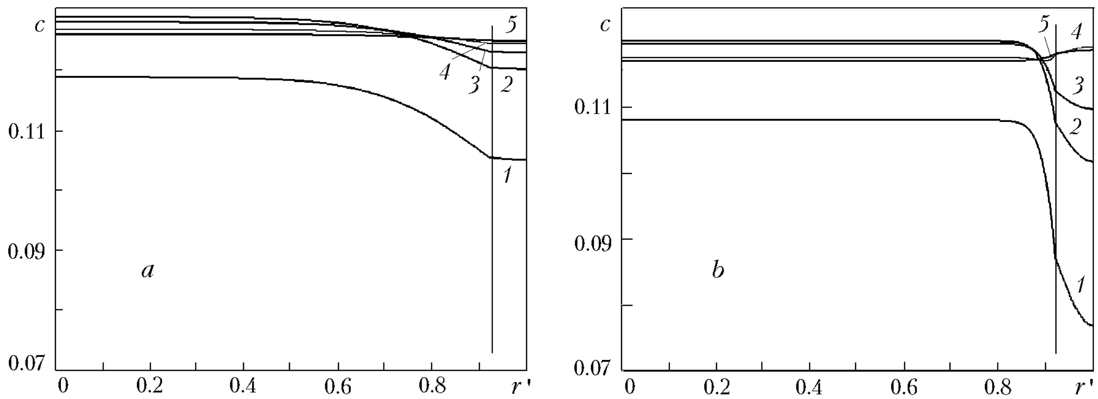

Fig. 3. Concentration of the impurity vs. radius at the height $x' = 0.5$ at different instants of time for Pe = 10 (a) and Pe = 125 (b): 1) $t' = 1.2$; 2) 2; 3) 2.5; 4) 3; 5) 4.

$$c_\infty = \frac{c_0}{1 + \frac{5.5}{m}\left((H_0')^{-0.18} - 1\right)}, \tag{44}$$

which follows from the equation of material balance of the tagged impurity. Figure 3 shows the concentration distributions of tagged particles on the radius of the riser for $x' = 0.5$. The influence of the coefficient of radial dispersion D_r on the dependences $c_1(r)$ and $c_2(r)$ is easily seen. Figure 4 shows a detailed evolution of the concentration fields with time. The propagation of the concentration wave of tagged particles in the bed's core (with a velocity u_1) is easily tracked. A special form of the concentration distribution of the particles at exit from the riser (Fig. 4h) corresponds to the condition $\bar{c}_1 = \bar{c}_2$ and reflects the influence of the internal circulation of the particles and the equality $c_2(t', 1, r') = \varphi(t')$. The rate of "washing-out" of the tagged impurity from the bottom bed is shown in Fig. 5. As is seen, the equilibrium concentration of tagged particles here is attained at $t' = 1.3$ (when Pe ≥ 1). Figure 6 compares the experimental data (calculated at $\Delta t \to \infty$) of [9], in which the values of $\bar{c}_1$ and $\bar{c}_2$ were measured at different points of a riser of diameter 0.305 m. The values of D_r found by the least-squares method lie in the range 0.0006–0.006 m^2/sec. It is noteworthy that the value D_r = 0.0025 m^2/sec obtained in [4] is in the interval indicated. Also noteworthy is the unusual kind of "tongues" (see Fig. 6b and c), reflecting the influence of internal circulation; this influence is particularly pronounced for small D_r (large Pe). The time of "arrival" of these "tongues" is calculated from the formula

$$t_d = \frac{H - H_0}{u_1} + \frac{H - x}{u_2}. \tag{45}$$

Relation (28) yields the expression for the coefficient β_1:

$$\beta_1 = -\frac{u_1}{\rho_1}\frac{d\rho_1}{dx}\frac{c_1|_{r_0}}{\bar{c}_1}. \tag{46}$$

In the longitudinal-mixing model [2], this parameter was expressed as follows:

$$\beta_1 = -\frac{u_1}{\rho_1}\frac{d\rho_1}{dx}. \tag{47}$$

In this connection, of interest is Fig. 7a, which gives the quantity $c_1|_{r_0}/\bar{c}_1$ calculated for different heights and instants of time. We can note that this ratio approaches unity only for relatively long times. Analogously for β_* we obtain from (29)

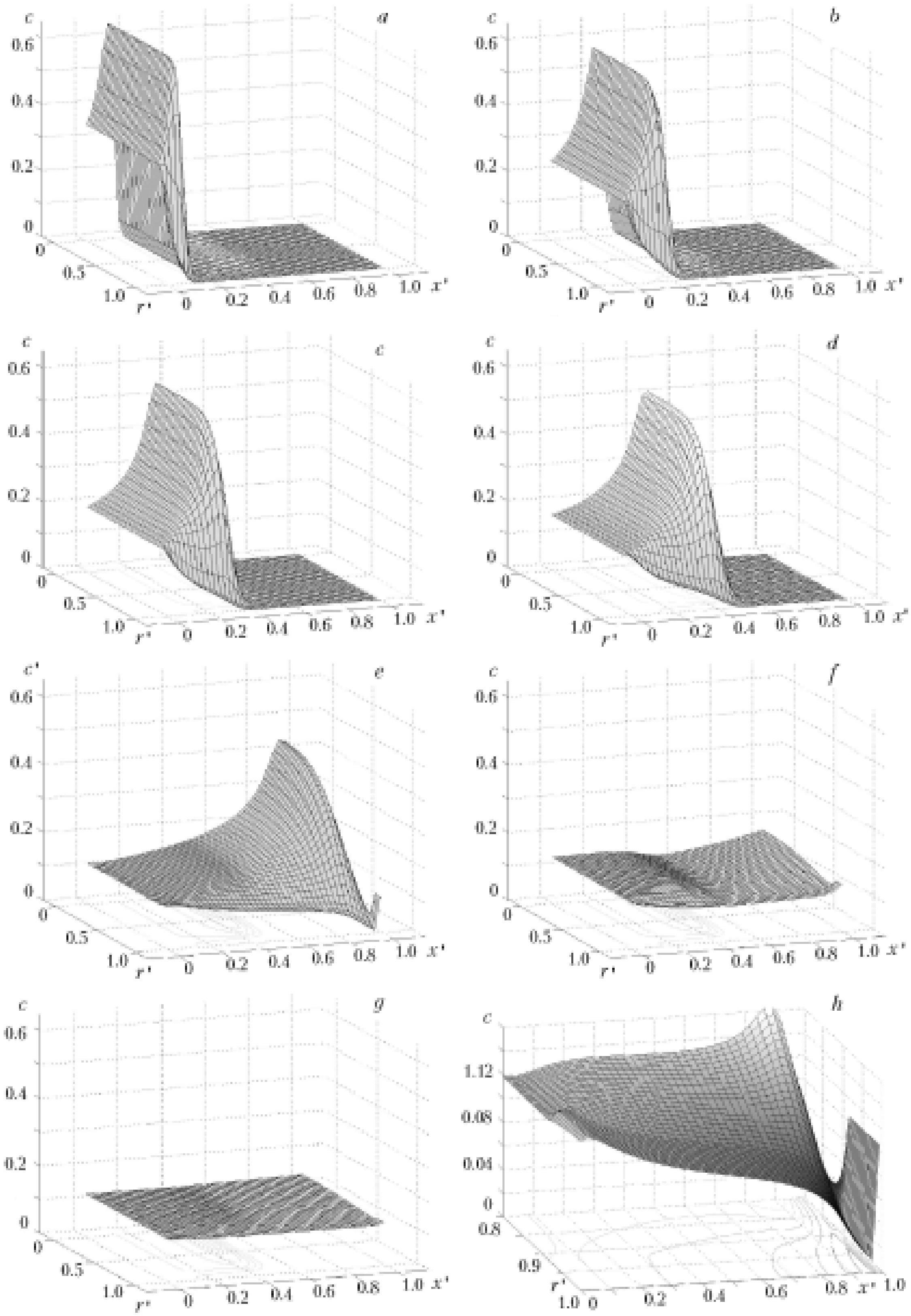

Fig. 4. Evolution of the concentration fields of the impurity [a) $t' = 0.1$; b) 0.2; c) 0.3; d) 0.4; e) 0.8; f) 1.2; g) 2] and the fragment of the field for the wall region at $t' = 0.8$ (h). Pe = 10.

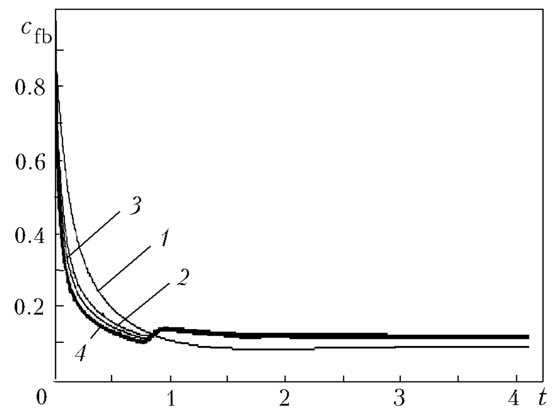

Fig. 5. Concentration of the impurity in the bottom bed vs. time: 1) Pe = 1; 2) 5; 3) 10; 4) 50, 500, 5000, and 50,000.

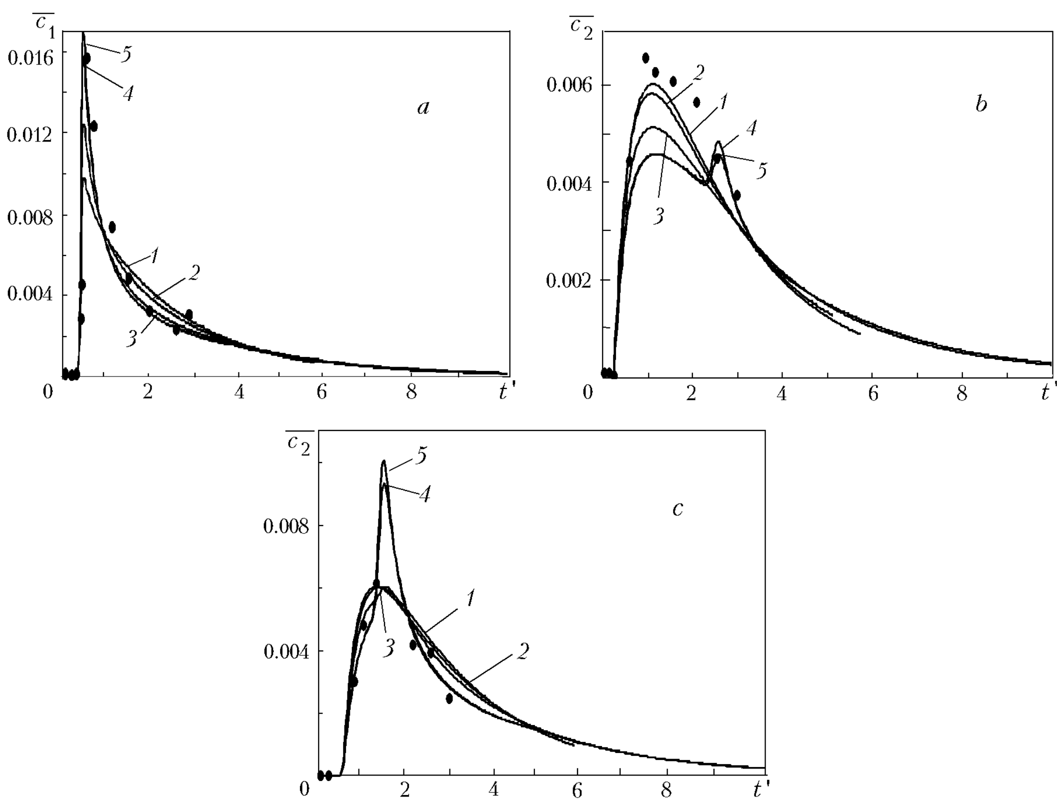

Fig. 6. Comparison of the calculated mixing curves to experimental data [9] at the height $x' = 0.55$ (a), $x' = 0.32$ (b), and $x' = 0.75$ (c): 1) Pe = 5; 2) 10; 3) 50; 4) 500; 5) 5000; points, experiment. $c_0 = 0.021$, $J_s = 147$ kg/(m^2·sec), $u = 4.57$ m/sec, and $H = 12.2$ m.

$$\beta_* = \frac{2D_r}{R^2}\frac{r_0\rho_1}{\rho}\left(\left.\frac{\partial c_1}{\partial r}\right|_{r_0} \Big/ (\bar{c}_2 - \bar{c}_1) \right) \tag{48}$$

or, in dimensionless form,

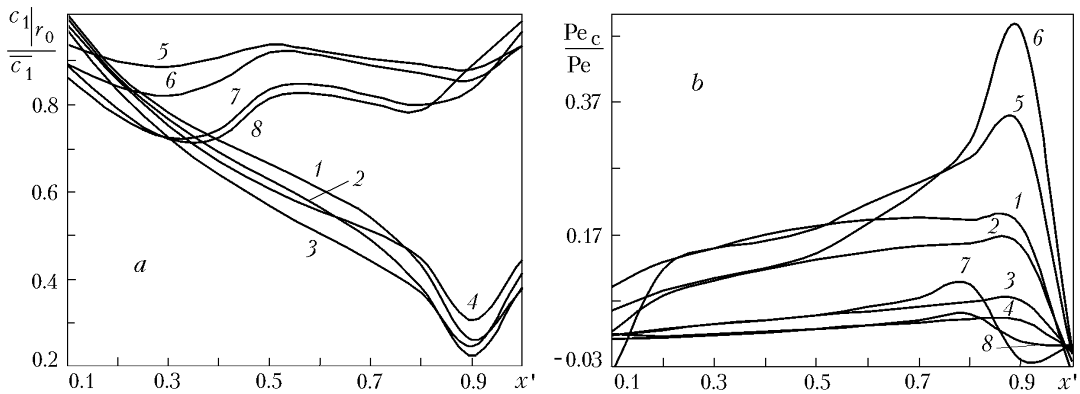

Fig. 7. Dependence of $c_1\big|_{r_0}/\bar{c}_1$ (a) and $\mathrm{Pe_c/Pe}$ (b) on the dimensionless height of the riser: 1 and 5) Pe = 5; 2 and 6) 10; 3 and 7) 50; 4 and 8) 125 [1–4) $t' = 0.8$; 5–8) 1.2].

$$\frac{\mathrm{Pe_c}}{\mathrm{Pe}} = \frac{A + Bu}{2\sqrt{A}} \frac{\bar{c}_2 - \bar{c}_1}{\left.\dfrac{\partial c'}{\partial r'}\right|_{\sqrt{A}}}, \tag{49}$$

where $\mathrm{Pe_c} = (u - u_t)/(\beta_* H)$. The dependence $\mathrm{Pe_c/Pe}$ on x' at different instants of time is given in Fig. 7b. As is seen, this quantity is not constant. Figure 7 demonstrates that model (1), (2) with constant coefficients β_* and β_1 is not, strictly speaking, a result of the averaging of (15) and (16). It represents an independent model that is capable, despite its simplicity, of describing experimental data satisfactorily, as the experience of using it in [2] has shown.

Thus, the formulated generalized mixing model allows for the basic features of the process: the presence of internal and external circulation of particles, convective particle fluxes in the radial direction, and the existence of the bottom bed. The model is capable of satisfactorily describing experimental data and yielding reasonable values of the coefficient of radial dispersion of particles.

This work was carried out with financial support from the Foundation for Basic Research of the Republic of Belarus, project No. T02R-071.

NOTATION

A, fraction of the riser's horizontal section occupied by ascending particles (bed's core); B, fraction of the riser's horizontal section occupied by descending particles (annular zone); c^*, concentration of particles, kg/m^3; c_1^* and c_2^*, concentrations of tagged particles in the bed's core and the annular zone, kg/m^3; $c = c^*/\rho(r)$, dimensionless concentration of particles; $c_1 = c_1^*/\rho_1$ and $c_2 = c_2^*/\rho_2$, dimensionless concentrations of tagged particles in the bed's core and the annular zones; $\bar{c}_1 = \frac{1}{r_0^2}\int_0^{r_0} 2c_1 r dr$ and $\bar{c}_2 = \frac{1}{R^2 - r_0^2}\int_{r_0}^{R} 2c_2 r dr$ bed's core and the annular zone; c_0, initial dimensionless concentration of tagged particles; D_a and D_r, coefficients of axial and radial dispersion of particles, m^2/sec; $\mathrm{Fr_t}$, Froude number, $\mathrm{Fr_t} = (u - u_t)^2/(gH)$; g, free-fall acceleration, m^2/sec; H, height of the riser, m; H_0, height of the bottom fluidized bed, m; $H_0' = H_0/H$; J_s, mass circulatory particle flux, kg/(m^2·sec); $\bar{J}_s$, dimensionless mass particle flux, $\bar{J}_s = J_s/\rho_s(u - u_t)$; $n = \rho_2/\rho_1$; Pe, Péclet numbers, $\mathrm{Pe} = (u - u_t)R^2/(HD_r)$; R, radius of the riser, m; r, radial coordinate, m; $r' = r/R$; r_0, radius of the bed's core, m; $r_0' = r_0/R = \sqrt{A}$; T, circulation period, sec; $T = \Delta t_r + \Delta t$; $T' = T(u - u_t)/H$; t, time; t', dimensionless time, $t' = t(u - u_t)/H$; Δt, recirculation time (time interval between the exit of tagged particles from the upper part of the riser to the entry into its base), sec; $\Delta t' = \Delta t(u - u_t)/H$; Δt_r, time in which particles in the bed's core traverse the riser's portion from $x = H_0$ to $x = H$, sec; u, rate of filtration of the gas, m/sec; u_t, free-fall velocity of a single particle, m/sec; u_1 and u_2, longitudinal velocities of particles in the bed's core and the annular zone, m/sec; u_1^* and u_2^*, radial velocities of particles in the bed's core and the annular zone, m/sec; $u_1' = u_1(u - u_t)$; $u_2' = u_2(u - u_t)$; $(u_1^*)' = u_1^*/(u - u_t)$; $(u_2^*)' = u_2^*/(u - u_t)$; v_s, particle velocity, m/sec; x, vertical coordinate, m; $x' = x/H$; β_*, interphase-exchange coefficient in (29), 1/sec; β_1, coefficient in (28), 1/sec; ε, porosity; $\rho = A\rho_1 + B\rho_2$, bed-density average over the horizontal section of the riser, kg/m^3; ρ_1 and ρ_2, bed densities in the core and the annular zone, kg/m^3. Subscripts and superscripts: a, axial; c, core of the transport zone; d, delay; fb, bottom fluidized bed; r, radial; s, particles; t, conditions of free fall of particles; 0, initial value; 1, bed's core; 2, bed's annular zone; bar, average value.

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